DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE.

Winter Examination – 2022

Course: - B. Tech. **Branch**: - Common for All branches **Semester:**- III

Subject Code & Name: BTBS301 **Engineering Mathematics-III**

Max. Marks: - 60 **Date: - 09/03/2023** **Duration:** - 3-Hrs

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Instructions to the Students:

1. *All the questions are compulsory.*
2. *The level of question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in () in front of the question.*
3. *Use of non-programmable scientific calculators is allowed.*
4. *Assume suitable data wherever necessary and mention it clearly.*

		(Level/CO)	Marks
Q. 1	**Solve Any Three of the following.**		12
A)	Find Laplace Transform of $e^{-3t}\sin^2 t$	L3/CO1	4
B)	Find Laplace Transform of $f(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$ where f(t) is periodic function of period 2.	L3/CO1	4
C)	Evaluate using Laplace Transform.: $\int_0^{\infty} \frac{\cos 4t - \cos 3t}{t}\, dt$	L3/CO1	4
D)	Find Laplace Transform of $(1 + 2t - 3t^2 + 4t^3)H(t-2)$	L3/CO1	4
Q2	**Solve Any Three of the following.**		12
A)	Find the inverse Laplace transformation of the function. $\log\left(1 + \frac{a^2}{s^2}\right)$	L3/CO2	4
B)	By using convolution theorem find $L^{-1}\left[\frac{s}{(s^2+4)(s^2+9)}\right]$	L3/CO2	4
C)	Find the inverse Laplace transformation of the function. $\frac{5s^2-15s-11}{(s+1)(s-2)^2}$	L3/CO2	4
D)	Solve using Laplace transformation $y'' + 3y' + 2y = t\delta(t-1)$ *for which* $y(0) = y'(0) = 0$	L3/CO2	4

Q.3 Solve Any Three of the following. (12)

A) Using Parseval's identity prove that $\int_0^\infty \frac{x^2}{(x^2+1)^2}dx = \frac{\pi}{4}$ L3/CO3 4

B) Find the Fourier transform of

$$f(x) = \begin{cases} 1-x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$ L3/CO3 4

C) Find the Fourier Sine transform e^{-ax}, $a > 0$ L3/CO3 4

D) Find the Fourier cosine transform of the function $f(y) = \begin{cases} \cos y, & 0 < y < a \\ 0, & y > a \end{cases}$ L3/CO3 4

Q.4 Solve Any Three of the following. (12)

A) Form the partial differential equation by eliminating arbitrary constants from L3/CO4 4

$$(x-a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$$

B) Solve the Partial differential equation $x(y-z)p + y(z-x)q = z(x-y)$ L3/CO4 4

C) Use the method of separation of variables to solve

$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ given that $u(x,0) = 6e^{-3x}$ L3/CO4 4

D) A bar with insulated at its ends is initially at temperature 0°C throughout. The end $x = 0$ is kept at 0 °C for all times and the heat is suddenly applied so that $\frac{\partial u}{\partial x} = 10 \; at \; x = t$ for all time. Find the temperature function $u(x,t)$ L3/CO4 4

Q.5 Solve Any Three of the following. (12)

A) Determine k such that the function $f(z) = e^x \cos y + ie^x \sin ky$ is analytic. L3/CO5 4

B) Show that $u = x^2 - y^2 - 2xy - 2x + 3y$ is a harmonic function and hence determine the analytic function $f(z)$ in terms of z. L3/CO5 4

C) Determine the pole of the function $f(z) = \frac{2Z-1}{Z(Z+1)(Z-3)}$ and also find the residue at each pole & sum of all residues. L3/CO5 4

D) Evaluate L3/CO5 4

$\oint_c \frac{\sin \pi z^2 + 2z}{(z-1)^2(z-2)}dz$, Where C is the circle $|z| = 4$

*** End ***

	DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE Supplementary Examination – Summer 2023 Course: B. Tech. (Common to all Branches) Semester : III Subject Name & Code: Engineering Mathematics – III (BTBSC 301) Max Marks: 60 Date:08/08/2023 Duration: 3 Hrs.	
	Instructions to the Students: 1. *Attempt any **FIVE** of the following questions.* 2. *All questions carry equal marks.* 3. *Use of non-programmable scientific calculators is allowed.* 4. *Assume suitable data wherever necessary and mention it clearly.*	
		Marks
Q. 1	**Solve Any Two of the following.**	**12**
A)	Find the Laplace transform of $F(t) = \frac{e^{-at}-e^{-bt}}{t}$	6
B)	Find the Laplace transform of $F(t) = \sin 2t \cos 3t$	6
C)	Find the Laplace transform of $erf\left(\sqrt{t}\right)$.	6
Q.2	**Solve Any Two of the following:**	**12**
A)	State and prove the convolution theorem for finding the inverse Laplace transform.	6
B)	Using Partial Fraction method, find the inverse Laplace transform of $\bar{f}(s) = \frac{5s+3}{(s-1)(s^2+2s+5)}$	6
C)	Find the inverse Laplace transform of $\bar{f}(s) = \cot^{-1}\left(\frac{s+3}{2}\right)$	6
Q. 3	**Solve any Two of the following:**	**12**
A)	Find the Fourier sine transform of $e^{-\|x\|}$, and hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi e^{-m}}{2}$, $m > 0$	6
B)	Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & \|x\| \le 1 \\ 0, & \|x\| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos\frac{x}{2} dx$.	6
C)	Evaluate the integral $\int_0^\infty \frac{t^2}{(t^2+1)^2} dt = \frac{\pi}{4}$.	6
Q.4	**Solve any Two of the following:**	**12**
A)	The partial differential equation by eliminating the arbitrary function from $z = x + y + f(xy)$	6
B)	The partial differential equations by eliminating the arbitrary constant $z = (x^2 + a)(y^2 + b)$	6
C)	Solve the following partial differential equations $(mz - ny)p + (nx + lz)q = ly - mx$ where the symbols have got their usual meanings.	6
Q. 5	**Solve any Two of the following:**	**12**
A)	Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is a harmonic function and hence determine the corre-	6

	sponding analytic function	
B)	If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant	6
C)	Under the transformation $W = \frac{1}{z}$, find the image of $\lvert z - 2i \rvert = 2$.	6
Q. 6	**Solve any Two of the following:**	
A)	Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$ along the path $y = x$ and $y = x^2$	6
B)	Evaluate $\oint_C \frac{e^{-z}}{z+1} dz$ where C is the circle $\lvert z \rvert = 2$ and $\lvert z \rvert = \frac{1}{2}$	6
C)	Use Cauchy's integral formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where C the circle is $\lvert z \rvert = 2$.	6
	*** End ***	

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End Semester Examination – Winter 2019

Course: B. Tech in	**Sem: III**
Subject Name: Engineering Mathematics-III (BTBSC301)	**Marks: 60**
Date: 10/12/2019	**Duration: 3 Hr.**

Instructions to the Students:

1. *Solve **ANY FIVE** questions out of the following.*
2. *The level question/expected answer as per OBE or the Course Outcome (CO) on which the question is based is mentioned in ( ) in front of the question.*
3. *Use of non-programmable scientific calculators is allowed.*
4. *Assume suitable data wherever necessary and mention it clearly.*

		(Level/CO)	Marks
Q. 1	**Attempt the following.**		12
A)	Find $L\left\{\cosh t \int_0^t e^u \cosh u \, du\right\}$	**Analysis**	4
B)	If $f(t) = \begin{cases} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases}$ is a periodic function with period 2π. Find $L\{f(t)\}$.	**Analysis**	4
C)	Using Laplace transform evaluate $\int_0^\infty e^{-at} \frac{\sin^2 t}{t} \, dt$	**Evaluation**	4
Q. 2	**Attempt any three of the following.**		12
A)	Using convolution theorem find $L^{-1}\left\{\frac{1}{s(s+1)(s+2)}\right\}$	**Application**	4
B)	Find $L^{-1}\{\bar{f}(s)\}$, where $\bar{f}(s) = \log\left(\frac{s^2+1}{s(s+1)}\right)$	**Analysis**	4
C)	Using Laplace transform solve $y'' + 2y' + 5y = e^{-t} \sin t$; $y(0) = 0$, $y'(0) = 1$	**Application**	4
D)	Find $L^{-1}\left\{\frac{s^2+2s-4}{(s-5)(s^2+9)}\right\}$	**Analysis**	4
Q. 3	**Attempt any three of the following.**		12

A)	Express the function $f(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$ as a Fourier sine integral and hence evaluate that $\int_0^\infty \frac{\sin\lambda x \sin\lambda\pi}{1-\lambda^2}\, d\lambda$.	Evaluation	4
B)	Using Parseval's identity for cosine transform, evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$.	Application	4
C)	Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 \le x \le 2. \\ 0, & x > 2 \end{cases}$	Analysis	4
D)	If $F_s\{f(x)\} = \frac{e^{-as}}{s}$, then find $f(x)$. Hence obtain the inverse Fourier sine transform of $\frac{1}{s}$.	Analysis	4
Q. 4	**Attempt any three of the following.**		12
A)	Form the partial differential equation by eliminating arbitrary function f from $f(x^2+y^2+z^2, 3x+5y+7z) = 0$	Synthesis	4
B)	Solve $pz - qz = z^2 + (x+y)^2$	Application	4
C)	Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ where the boundary conditions are $u(0,t) = 0$, $u(l,t) = 0$ $(t > 0)$ and the initial condition $u(x,0) = x$; l being the length of the bar.	Analysis	4
D	Use the method of separation of variables to solve the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given that $u(x,0) = 6e^{-3x}$	Application	4
Q. 5	**Attempt the following.**		12
A)	Determine the analytic function $f(z)$ in terms of z whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$	Analysis	4
B)	Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic.	Analysis	4
C)	Find the bilinear transformation which maps the points $z = 0, -1, -i$ onto the points $w = i, 0, \infty$. Also, find the image of the unit circle $\|z\| = 1$.	Analysis	4
Q. 6	**Attempt the following.**		12

A)	Use Cauchy's integral formula to evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\, dz$, where C is the circle $\lvert z \rvert = 3$.	Evaluation	4
B)	Find the poles of function $\frac{z^2-2z}{(z+1)^2(z^2+4)}$. Also find the residue at each pole.	Analysis	4
C)	Evaluate $\oint_C \frac{e^z}{\cos \pi z}\, dz$, where C is the unit circle $\lvert z \rvert = 1$.	Evaluation	4
	*** **Paper End** ***		

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

End Semester Examination – Winter 2018

Course: S.Y.B. Tech (All Branches) **Semester:** III

Subject Name: Engineering Mathematics-III **Subject Code:** BTBSC301

Max Marks:60 **Date:**30/11/2018 **Duration:** 03 Hrs

Instructions to the Students:

1. *Attempt **Any Five** questions of the following. All questions carry equal marks.*
2. *Use of non-programmable scientific calculators is allowed.*
3. *Figures to the right indicate full **Marks**.*

Q. 1. a) Show that,

$$\int_0^\infty \frac{\sin at}{t}\,dt = \frac{\pi}{2}.$$ [4]

b) Find the Laplace transform of

$$\int_0^t \frac{e^{-3u}\sin 2u}{u}\,du.$$ [4]

c) Find the Laplace transform of the function

$$f(t) = \begin{cases} 2 & ,0 < t < \pi \\ 0 & ,\pi < t < 2\pi \\ \sin t & , \quad t > 2\pi \end{cases}$$ [4]

Q.2 . a) Find the inverse Laplace transform of $\cot^{-1}\left(\frac{s+3}{2}\right)$. [4]

b) By convolution theorem, find inverse Laplace transform of

$$\frac{s}{(s^2+1)(s^2+4)}.$$ [4]

c) By Laplace transform method, solve the following simultaneous equations

$$\frac{dx}{dt} - y = e^t;\ \frac{dy}{dt} + x = \sin t;\ \ given\ that\ \ x(0) = 1, y(0) = 0.$$ [4]

Q. 3. a) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0 & , \ |x| > 1. \end{cases}$$ [4]

b) Find the Fourier sine transform of $e^{-|x|}$, and hence show that

$$\int_0^\infty \frac{x\sin mx}{1+x^2}\,dx = \frac{\pi e^{-m}}{2}, \quad m > 0.$$ [4]

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c) Using Parseval's Identity, prove that

$$\int_0^{\infty} \frac{t^2}{(t^2+1)^2}\, dt = \frac{\pi}{4}.$$ [4]

Q.4. a) Solve the partial differential equation [4]

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy.$$

b) Use method of separation of variables to solve the equation

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \text{ ; given that } u(x,0) = 6e^{-3x}.$$ [4]

c) Find the temperature in bar of length 2 *units* whose ends are kept at zero temperature and lateral surface insulated if initial temperature is [4]

$$\sin\left(\frac{\pi x}{2}\right) + 3\sin\left(\frac{5\pi x}{2}\right).$$

Q. 5. a) If $f(z)$ is analytic function with constant modulus, show that $f(z)$ is constant. [4]

b) If the stream function of an electrostatic field is $\psi = 3xy^2 - x^3$, find the potential function ϕ, where $f(z) = \phi + i\psi$. [4]

c) Prove that the inversion transformation maps a circle in the z-plane into a circle in w-plane or to a straight line if the circle in the z-plane passes through the origin. [4]

Q.6. a) Evaluate $\oint_c \frac{e^z}{(z-2)}\, dz$, where c is the circle $|z| = 3$. [4]

b) Evaluate $\oint_c \tan z\, dz$, where c is the circle $|z| = 2$. [4]

c) Evaluate , using Cauchy's integral formula : [4]

1) $\oint_C \frac{\cos(\pi z)}{(z^2-1)}\, dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$.

2) $\oint_C \frac{\sin^2 z}{(z-\frac{\pi}{6})^3}\, dz$, where C is the circle $|z| = 1$.

*** End ***